RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2018

THIRD YEAR [BATCH 2016-19]

Date : 15/12/2018 **PHYSICS (Honours)**

Time : 11.00 am – 1.00 pm **Paper : V [Gr. A&B]** Full Marks : 50

(Use a separate Answer Book for each group)

Group – A

Answer **any three** questions from the following: [3 × 10]

1. a) Define holonomic constraint and virtual displacement. State and explain the principle of virtual work for a constrained dynamical system. 3

   b) Hence obtain D'Alembert's principle starting from Newton's equation of motion for a constrained N-particle system. 2

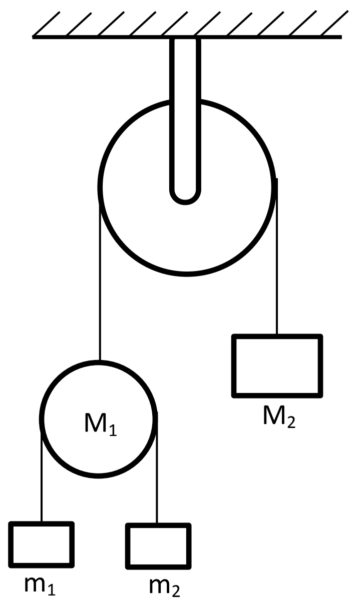
   c) Using the concept of generalized coordinates, deduce Lagrange's equations of motion from D'Alembert's principle, in terms of the Kinetic energy *T*. 5

2. a) Explain briefly why Newtonian concept of momentum has to be revised for a charge particle moving under the generalised potential *U* of electromagnetic field, where $U = e\phi - e\left(\vec{v}\cdot\vec{A}\right)$.

   [Note that *e* = charge of the particle, ϕ is the scalar potential, $\vec{A}$ is the vector potential and $\vec{v}$ is the velocity of the charged particle.] 2

   b) What are cyclic coordinates? How do their existence in a Lagrangian lead to conservation principles. Explain with an example. 3

   c) A mass M_2 hangs at one end of a string which passes over a fixed frictionless non-rotating pulley (see figure). At the other end of this string there is another non-rotating pulley of mass M_1 over which there is a string carrying masses m_1 and m_2. (a) Set up the Lagrangian of the system. (b) Find the acceleration of mass M_2. 5



3. a) Derive the Hamilton's canonical equation of motion by applying Legendre's dual transformation to the Lagrangian of a system. 3

b) The Lagrangian of an anharmonic oscillator is $L(x,\dot{x}) = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega^2 x^2 - \alpha x^3 + \beta x\dot{x}^2$.

Obtain the Hamiltonian of the system and find the equation of motion. 2+2

c) State and prove Jacobi Poisson theorem on Poisson Brackets. What is the significance of this theorem. 3

4. a) Show that the Lagrangian in generalised coordinates for small oscillation about all equilibrium position, can be written as $L = \frac{1}{2}\left(T_{ij}\dot{q}_i\dot{q}_j - V_{ij}q_i q_j\right)$, where $T_{ij} = \left(\frac{\partial^2 T}{\partial \dot{q}_i \partial \dot{q}_j}\right)_0$ and $V_{ij} = \left(\frac{\partial^2 V}{\partial q_i \partial q_j}\right)_0$ are $n\times n$ symmetric matrices and n is the number of degree of freedom of the system. 4

b) Explain briefly how you can solve the coupled equations obtained from the above Lagrangian. 2

c) Determine the normal mode frequency of the Lagrangian, given by $L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}\left(\omega_1^2 x^2 + \omega_2^2 y^2\right) + \alpha xy$. 4

5. a) Derive the Euler's equations for the motion of a rigid body with one point fixed in space under the action of a torque. 3

b) Solve the Euler's equations for the force-free motion of a symmetrical top for which $I_1 = I_2$ and I_3 is the moment of inertia about the axes of symmetry of the body. What do you mean by precessional motion in this case? 4+1

c) Determine the precessional period at the Earth, given the polar diameter of Earth is 12710 km and the equatorial diameter is 12754 km. 2

Group – B

Answer **any two** questions from the following: [2 × 10]

6. a) Write down the Lorentz transformation between two inertial frames which have a constant relative velocity v between them along their common 'X' axis. The frames have their clocks synchronised to zero when their origins are coincident. 2

b) Show that $s^2 = c^2t^2 - x^2 - y^2 - z^2$ is invariant under the Lorentz transformations defined in 6(a). Use this to deduce the formula of time dilation. 4

c) A rod of proper length l_0 oriented at an angle θ with respect to the X axis moves with velocity u along the 'X' axis in frame S. What is the length measured by an observer S' which is moving along the 'X' axis with a velocity v with respect to the frame S. 4

7 a) An astronaut wants to go to a planet 15 light years away from the earth. The rocket accelerates quickly (the time for which can be neglected) and then moves at a uniform velocity. With what velocity the rocket must move relative to the earth if the astronaut is to reach there in one year as measured by clocks at rest on the rocket. 4

b) Two clocks are positioned at the ends of a train of proper length L. They are synchronized in the train frame. The train travels at a uniform velocity v past a platform. If the clocks are looked at simultaneously from the platform, will they show the same time? If not, which clock shows a higher reading and by how much? 3

c) A train with proper length L moves at speed $\frac{5c}{13}$ with respect to the ground. A ball is thrown inside the train from the back to the front. The speed of the ball with respect to the train is $\frac{c}{3}$. As viewed by someone on the ground, how much time does the ball spend in the air and how far does it travel? 3

8. a) How do you define a 4 vector under Lorentz transformation? Find the scalar product of the 4 velocity with itself for a particle moving with velocity $\bar{u}$. Find the 4 components of the momentum 4-vector. 3

b) Show that the scalar product of force and velocity 4-vectors is zero. 3

c) Using 4-vector formulation, show that the rest energy of a particle of mass m is given by $E_0 = mc^2$ and in general $E^2 = c^2p^2 + m^2c^4$. 4

9. a) Show that the kinetic energy of a particle of mass m moving with a velocity v can be expressed as $T = \frac{\gamma^2\beta^2}{\gamma+1}mc^2$ where $\beta = v/c$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. 4

b) Using the transformation law of 4-vector momentum, find the longitudinal Doppler shift in frequency υ of a source of light receding from the observer with a velocity v. 3

c) Distinguish between time like, space like and light like 4-vectors. Show that if two events have a time like interval, one can always find an inertial frame where the events are co-local, i.e. coincident. 3

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